

Physics 129a

Solutions to Midterm #2

$$1. \quad \frac{F_2^P(x)}{x} = \frac{4}{a} [u(x) + \bar{u}(x)] + \frac{1}{a} [d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]$$

Using $u^n(x) = d^P(x)$ and $d^n(x) = u^P(x)$

$$\frac{F_2^n(x)}{x} = \frac{4}{a} [d(x) + \bar{d}(x)] + \frac{1}{a} [u(x) + \bar{u}(x) + s(x) + \bar{s}(x)]$$

$$\therefore F_2^P(x) - F_2^n(x) = \frac{x}{3} [u(x) + \bar{u}(x) - d(x) - \bar{d}(x)]$$

writing $u(x) = u_{\text{valence}}(x) + u_{\text{sea}}(x)$ $\bar{u}(x) = \bar{u}_{\text{sea}}(x)$
 $d(x) = d_{\text{valence}}(x) + d_{\text{sea}}(x)$ $\bar{d}(x) = \bar{d}_{\text{sea}}(x)$

$$\begin{aligned} F_2^P(x) - F_2^n(x) &= \frac{x}{3} [u_{\text{valence}}(x) + u_{\text{sea}}(x) + \bar{u}_{\text{sea}}(x) \\ &\quad - d_{\text{valence}}(x) + d_{\text{sea}}(x) + \bar{d}_{\text{sea}}(x)] \\ &= \frac{x}{3} [u_{\text{valence}}(x) - d_{\text{valence}}(x)] \end{aligned}$$

$$\int_0^1 [F_2^P(x) - F_2^n(x)] dx = \int_0^1 \frac{x}{3} [u_{\text{valence}}(x) - d_{\text{valence}}(x)] dx$$

But we are told that $u_{\text{valence}}(x)$ & $d_{\text{valence}}(x)$ have the same shape.

$$\int u_{\text{valence}}(x) dx = 2$$

$$\int d_{\text{valence}}(x) dx = 1$$

$$\Rightarrow \frac{u_{\text{valence}}(x)}{d_{\text{valence}}(x)} = 2$$

[Since the shapes are the same in this problem]

$$1/\text{cont} \quad \therefore \int_0^1 F_2^P(x) - F_2^N(x) dx = \frac{1}{3} \int x \left(u_{val}(x) - \frac{u_{val}(x)}{2} \right) dx$$

$$\therefore \frac{1}{6} \int_0^1 x u_{val}(x) dx = 0.21 - 0.15 = 0.06$$

$$\Rightarrow \int_0^1 x u_{val}(x) dx = \boxed{0.36}$$

$$b. \int F_2^P(x) dx = \int \frac{4}{9} [u_{val}(x) + u_{sen}(x) + \bar{u}_{sen}(x)]$$

$$+ \frac{1}{9} [d_{val}(x) + d_{sen}(x) + \bar{d}_{sen}(x) + \bar{s}_{sen}(x)] x dx$$

using $d_{val}(x) = \frac{1}{2} u_{val}(x)$
 and $u_{sen}(x) = \bar{u}_{sen}(x) = d_{sen}(x) = \bar{d}_{sen}(x) =$
 $\bar{s}_{sen}(x) = \bar{s}_{sen}(x)$

$$\int F_2^P(x) dx = \int \left[\left(\frac{4}{9} + \frac{1}{18} \right) u_{val}(x) + \left(\frac{8}{9} + \frac{4}{9} \right) u_{sen}(x) \right] x dx$$

$$0.21 = \frac{9}{18} \int x u_{val}(x) dx + \frac{12}{9} \int x u_{sen}(x) dx$$

$$= \frac{9}{18} (0.36) + \frac{4}{3} \int x u_{sen}(x) dx$$

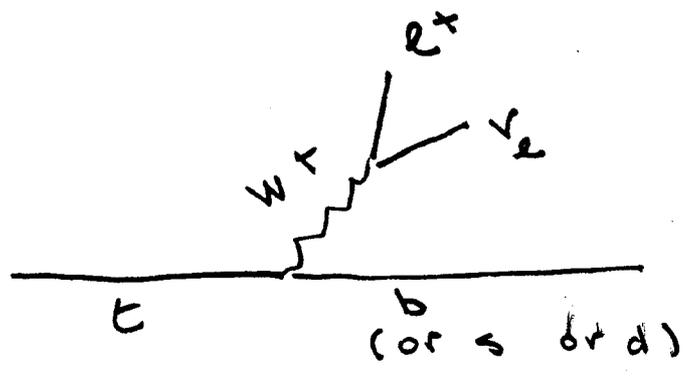
$$\therefore \int x u_{sen}(x) dx = \frac{3}{4} (0.21 - 0.18) = \frac{0.09}{4}$$

$$= 0.0225$$

$$\int x (\bar{u}(x) + \bar{d}(x) + \bar{s}(x)) dx = 3 \int x u_{sen}(x) dx$$

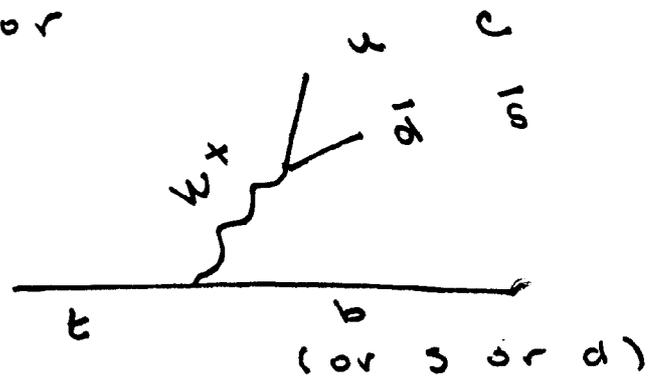
$$= \boxed{0.0675}$$

2a) $t \rightarrow b, s \text{ or } d$ by emitting a W^+ . But $V_{tb} = 0.999$ so s and d modes are very small



$l = e, \mu, \tau$

or

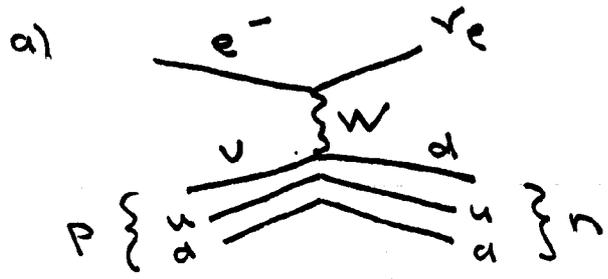


b The decays involving quarks have a color factor of 3. All the quarks and leptons have masses that are small compared to the t mass, so the difference in phase space can be neglected.

$$\therefore BR(t \rightarrow X \mu^+ \nu_\mu) = \frac{1}{\underbrace{1+1+1}_{3 \text{ lepton decays}} + \underbrace{3+3}_{2 \text{ quark decays} \times 3 \text{ colors}}} = \frac{1}{9}$$

2c $\frac{\tau_t}{\tau_\mu} = \frac{m_\mu^5}{m_t^5} = \left(\frac{0.105}{175}\right)^5 = 7.8 \times 10^{-17}$
 $\tau_t = (2.2 \times 10^{-6} \text{ s})(7.8 \times 10^{-17}) = \boxed{1.7 \times 10^{-22} \text{ s}}$

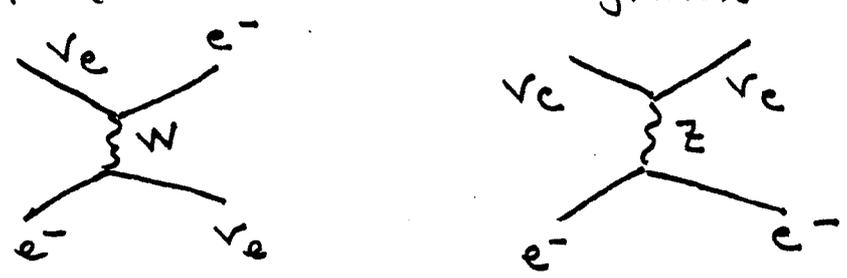
3. $e^- + p \rightarrow n + \nu_e$



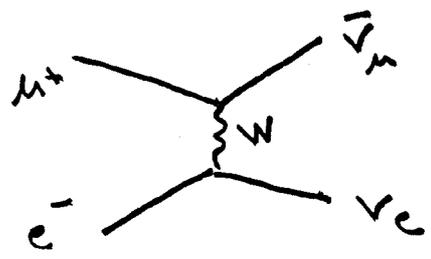
(charge of W
for internal lines
not uniquely define)

b) $\nu_e + e^- \rightarrow \nu_e + e^-$

There are 2 diagrams:



c) $\mu^+ + e^- \rightarrow \bar{\nu}_\mu + \nu_e$



d) $e^+e^- \rightarrow \mu^+ \mu^-$

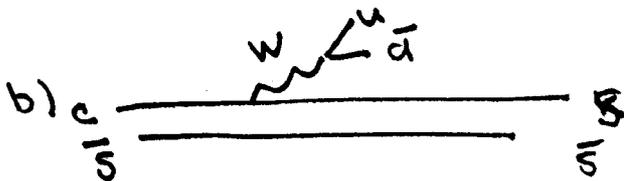
There are 2 diagrams:



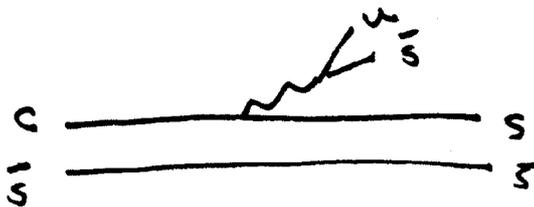
4. a) $R = \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+ \mu^-}$

The top occurs via $e^+e^- \rightarrow q\bar{q}$
 For 25 GeV, q can be u, d, s, c, b

$$R = \underset{\substack{\uparrow \\ \text{color}}}{3} \sum q^2 = 3 \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} \right) = 11/3$$



$D_s^+ \rightarrow \pi^+ \phi$



$D_s^+ \rightarrow K^+ \phi$

The top diagram goes like $|V_{cs} V_{ud}|^2$
 the bottom goes like $|V_{cs} V_{us}|^2$

$$\therefore \frac{\Gamma(D_s \rightarrow \phi K^+)}{\Gamma(D_s \rightarrow \phi \pi^+)} \sim \left| \frac{V_{us}}{V_{ud}} \right|^2 = \left(\frac{0.224}{0.975} \right)^2 = 0.051$$

c) Y is a $b\bar{b}$ bound state
 ψ is a $c\bar{c}$ bound state

Both decays go via virtual g :



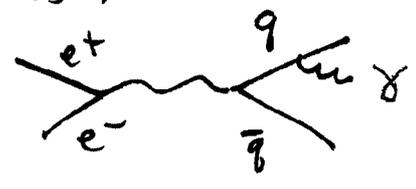
$$\Gamma \sim 16 \pi \alpha^2 \frac{Q^2}{M_V^2} |\psi(0)|^2$$

But $|\psi(0)|^2 / M_V^2 = \text{const}$
 So rate goes like Q^2

$$\therefore \frac{\Gamma(Y \rightarrow \mu^+ \mu^-)}{\Gamma(\psi \rightarrow \mu^+ \mu^-)} = \frac{(1/3)^2}{(2/3)^2} = \frac{1}{4}$$

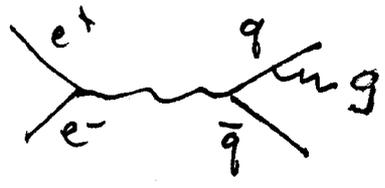
$$a) \frac{\sigma(e^+e^- \rightarrow 2 \text{ jets} + g)}{\sigma(e^+e^- \rightarrow 3 \text{ jets})}$$

Top diagram



(Plus sum off other leg)

Bottom diagram



(Plus sum off other leg)

only difference is coupling constant α rather than $\frac{4}{3} \alpha_s$ (the $\frac{4}{3}$ is a color factor)

$$\therefore \text{ratio} = \frac{1/129}{\frac{4}{3}(0.1)} = 0.058 \quad (\text{where } \alpha = \frac{1}{129} \text{ at the } \dots)$$